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Application of 2-Dominator Coloring in Graphs Using MATLAB

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ABSTRACT

A graph has a dominator coloring if it has a proper coloring in which each vertex of the graph dominates every vertex of some color class. New bounds on the dominator coloring number of a graph are presented, with applications to circulant, cayley and bucky ball graphs. A two dominator coloring of graph with minimum degree atleast one is a proper coloring of graphs with the extra property that every vertex in the graph properly dominates a two color class.

Keywords: Two Dominator Coloring, Circulant Graph, Cayley Graph.

1. INTRODUCTION

Let G= (V,E) be a graph of order p with a minimum degree at least one. The open neighborhood N(v) of a vertex v ∈V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v]=N(v)∪{v}.For a set S⊆V, the open neighborhood N(S) is defined to be $\cup_{v\in s} N(v)$, and the closed neighborhood of S is N[S]=N(S)∪S. A subset S of V is called a dominating (total dominating) set if every vertex in V-S (V) is adjacent to some vertex in S. A dominating (2-dominating) set is minimal dominating (2-dominating) set if no proper subset of S is a dominating (2- dominating) set of G. The domination number γ (2-domination number γ_t) is the minimum cardinality taken over all two dominating sets of G.A γ set (γ_t set) is any 2-dominating set with cardinality (t).

A proper coloring of G is an assignment of colors to the vertices of G, such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ (G).

A dominator coloring on graphs is a proper coloring of graphs with the extra property that every vertex in the graph dominates an entire color class. The smallest number of colors for which there exists a dominator coloring of G is called dominator chromatic number of G and is denoted by χ_d(G). This concept was introduced by Raluca Gera *et al*.[1].

In this a new concept for 2-dominator coloring $\chi_{d,2}(G)$ in G is introduced. Also we introduce the application for some graphs in graph theory is introduced.

2. CIRCULANT GRAPH

In graph theory, a circulant graph is an undirected graph that has a cyclic group of symmetries that include a symmetry taking any vertex to any other vertex.

2.1 Definition

A circulant graph is a graph which has a circulant adjacency matrix. A circulant graph of order n has vertex set V (G) = Zn and edge set E(G) = {uv : u − v ∈ S}, for some generating set S ⊆ V (G). This set S must not contain the identity element 0, and must be closed under additive inverses. It is said that $C_{n,s}$ is the circulant graph of order n with generating set S.

It is noted note that $C_{n,s}$ is an undirected Cayley graph for the group G = (Zn, +). Thus, circulant graphs are a special case of the more general family of Cayley graphs. Since our generating set S must be closed under additive inverses and does not contain the identity element, the following is an equivalent definition of $C_{n,s}$.

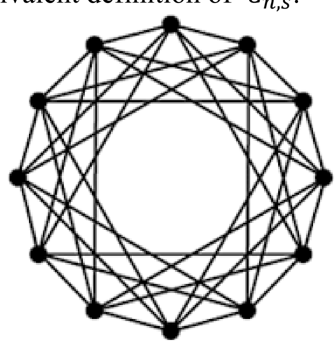

Fig 2.1 Circulant Graph

Application for Circulant Graph

There are many applications of the theory of circulants. Indeed, many researchers have independently rediscovered circulants for this reason. Sometimes a researcher would be unaware of the name "circulant"; indeed one common alternative name was "cyclic matrix."

Circulant graphs have a vast number of uses and applications to telecommunication network, VLSI design, parallel and distributed computing. The applications are mainly in pure mathematics and technology which mysteriously reflects the abstract concrete dichotomy of the theory of circulants. For instance, modern telecommunications would be impossible without frequency analysis. With the advent of fast digital computing, the main technique of frequency analysis has become the discrete Fourier transform. This transform is also the single most important transform on circulants, so much so, that much of the theory of circulants can be regarded as the theory of the discrete Fourier transform. Circulants are important in digital encoding; this is a wondrous technology it enables devices ranging from computers to music players to recover from errors in transmission and storage of data, and restore the original data. However, the initial impetus to the study of circulants was not technological but rather stemmed from problems in pure mathematics, particularly number theory. Circulant graphs are an important sub-field of graph theory. As we proceed, we shall point out applications of circulants to homogeneous diophantine equations and combinatorial analysis. Finally, towards the end of the application, a return is possible to the physical sciences with an application of circulants to the evolution of density fluctuations.

3. CAYLEY GRAPH

Cayley graphs give a way of encoding information about group in a graph. Given a group with a, typically finite, generating set, we can form a Cayley Graph for that group with respect to that generating set.

Let β =<B,.> be a group with generating set X. The fiber over a generator x ∈ X in the Cayley digraph C~ (β, X) or Cayley graph C(β, X) is the set x˜ = $\{x_b$ | b ∈ B}

Terminology Note The traditional way to draw a Cayley digraph C~ (B, X) labels the vertices by group elements. Edges were not given distinct names. Instead, a different color or graphic feature was used for each edge fiber ˜x, which led to the terminology Cayley color graph.

Application for Cayley Graph

Cryptology is the scientific study of writing secret information. Throughout history, people have been interested in keeping information secret from certain people and people have been interested in reading secret information that is being kept from them. Cryptology has two main branches: cryptography, the study of designing methods for hiding information (generally, from all but a select group of people), and crypt analysis, the study of methods for getting at hidden information.

The most fundamental cryptographic problem is designing a pair of algorithms (called an encryption scheme): one for turning regular text into secret text (called an encryption algorithm) and one for undoing the encryption, turning secret text back into its original text (this algorithm is called a decryption algorithm). These algorithms should have the property that the decryption can only be done by people whom the sender (the person doing the encrypting) intended to be able to read it (the receivers).

A classical example of such an algorithm is the shift cipher: before sending any messages, the sender somehow shares a number between 1 and 25 (represent this number by the variable k) with the receivers. Then, the encryption algorithm simply takes the message, letter by letter, and shifts it k letters forward in the alphabet (wrapping around from z to a). The decryption algorithm simply shifts backward by k. For example, if Alice and her friend Bob both shared k = 3, Alice would encrypt the message attack dawn as dwwdfndwgdzq. Bob, knowing k = 3, could decrypt the message and learn when Alice wishes to attack. Additionally a third party who doesn't know the value of K, will simply see dwwdfndwgdzq.

4. BUCKY BALL

Definition

One interesting construction for graph analysis is the Bucky ball. This is composed of 60 points distributed on the surface of a sphere in such a way that the distance from any point to its nearest neighbors is the same for all the points. Each point has exactly three neighbors. Among these, the number of hexagons varies from one type of fullerene to another but every fullerene has exactly twelve pentagons. This is not an accident. It is a consequence of a theorem of the great eighteenth century mathematician Leonhardt Euler. A famous formula of Euler, perhaps the first formula in topology, says the following: The surface of a polyhedron in three dimensional space is made up of (two dimensional) faces, (one dimensional) edges, and (zero dimensional) vertices. Let f denote the number of faces, e the number of edges, and v the number of vertices. Then Euler's formula says $f - e + v = 2$. For example, the surface of a cube has six (square) faces, twelve edges, and eight vertices and $6 - 12 + 8 = 2$. The regular icosahedron has twelve vertices and twenty (triangular) faces. Five edges emanate from each vertex, but each edge impinges on two vertices. So there are thirty edges ($5\times12/2$). Once again, $20 - 30 + 12 = 2$. Suppose we truncate the icosahedron at one of its vertices, as in the figure, this has the effect of replacing the vertex by a pentagonal face. One face is added, deleted one vertex, and added five new edges and five new vertices. This clearly does not change the value of $f - e + v$, so it remains two. If we do this in a symmetric fashion at all the vertices, the buckyball is obtained.

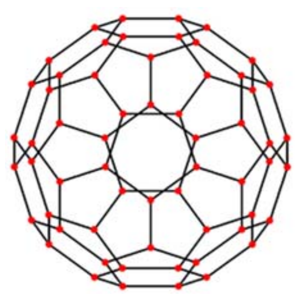

Fig 3.1 Bucky ball

Application

Fullerenes have been extensively used for several biomedical applications including the design of high-performance MRI contrast agents, X-Ray imaging contrast agents, photodynamic therapy and drug and gene delivery, summarized in several comprehensive reviews.

While past cancer research has involved radiation therapy, photodynamic therapy is important to study because breakthroughs in treatments for tumor cells will give more options to patients with different conditions. More recent experiments using HeLa cells in cancer research involves the development of new photo sensitizers with increased ability to be absorbed by cancer cells and still trigger cell death. It is also important that a new photosensitizer does not stay in the body for a long time to prevent unwanted cell damage[58].

Fullerenes can be made to be absorbed by HeLa cells. The C60 derivatives can be delivered to the cells by using the functional groups L-phenylalanine, folic acid, and L-arginine among others[59]. The purpose for functionalizing the fullerenes is to increase the solubility of the molecule by the cancer cells. Cancer cells take up these molecules at an increased rate because of an up regulation of transporters in the cancer cell. In this case amino acid transporters will bring in the L-arginine and L-phenylalanine functional groups of the fullerenes.

Once absorbed by the cells, the C60 derivatives would react to light radiation by turning molecular oxygen into reactive oxygen which triggers apoptosis in the HeLa cells and other cancer cells that can absorb the fullerene molecule. This research shows that a reactive substance can target cancer cells and then be triggered by light radiation, minimizing damage to surrounding tissues while undergoing treatment.

When absorbed by cancer cells and exposed to light radiation, the reaction that creates reactive oxygen damages the DNA, proteins, and lipids that make up the cancer cell. This cellular damage forces the cancerous cell to go through apoptosis, which can lead to the reduction in size of a tumor. Once the light radiation treatment is finished the fullerene will reabsorb the free radicals to prevent damage of other tissues. Since this treatment focuses on cancer cells, it is a good option for patients whose cancer cells are within the reach of light radiation. As this research continues into the future, it will be able to penetrate deeper into the body and be absorbed by cancer cells more effectively.

SIMULATION RESULT AND DISCUSSION

A two dominator coloring of graph theory will be simulated using MATLAB. Here we simulate the circulant, cayley and bucky ball graphs in graph theory.

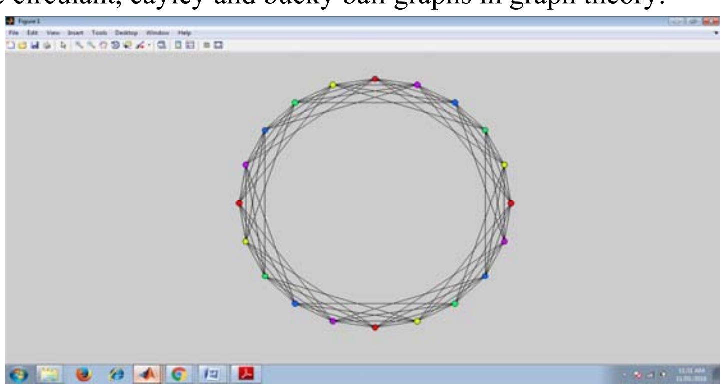

Fig 4.1 Simulation for Circulant Graph

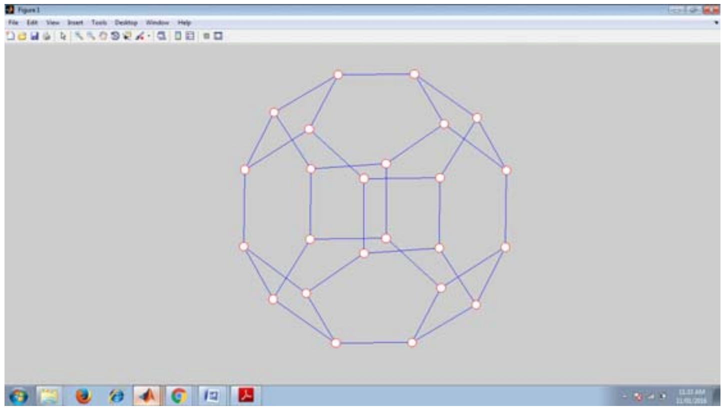

Fig 4.2 Simulation for Cayley graph

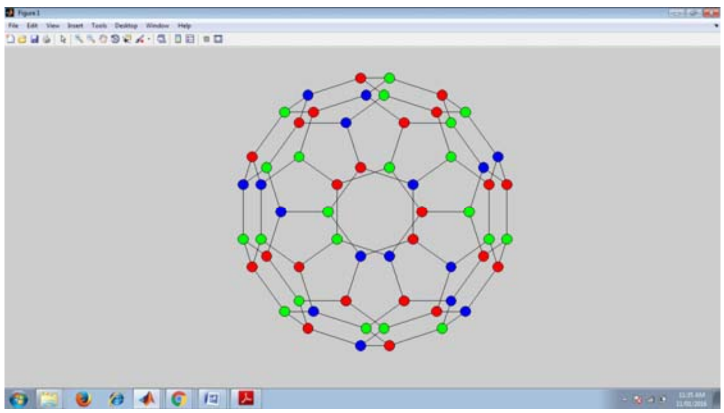

Fig 4.3 Simulation for Bucky ball graph

CONCLUSION

In this paper, application of 2-dominator coloring is introduced and derived some results in 2-dominator coloring of circulant graphs, cayley and bucky ball graph and also several results on this coloring parameter are obtained. This will be an initiative for the study of k- dominator coloring.

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