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A Fixed Point Theorem in Hausdorff Space

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ABSTRACT

The aim of this paper is to prove the fixed point theorem in Hausdorff space.

Keywords: Fixed Point, Hausdorff space, continuous mapping.

1. INTRODUCTION

In the past few years, a number of authors such as Singh and zarzitto[1] Ray and Chatterjee[2], Chatterjee and Ghoshal[3] Fisher and Khan[4] and Popa[5] etc. have established several interesting results on fixed point in different types.

2. MAIN RESULTS

In this paper we prove the following theorem in Hausdorff space.

Theorem: Let T be a continuous mapping of a Hausdorff space X into itself and let
F : XxX→ [0,α) be a countinuous mapping such that for each pair of distinct points x,y ε X.

$$F(Tx,Ty) \le \alpha \frac{F(x,Tx)F(y,Ty)}{F(x,Ty)+F(y,Tx)+F(x,y)} + \beta \frac{F(x,y)[1+\sqrt{F(x,y)F(x,Tx)}+\sqrt{F(x,y)F(y,Tx)}]}{[1+F(x,y)+F(x,Tx)F(x,Ty)F(y,Tx)F(y,Ty)]} + \gamma \frac{F(y,Ty)[1+\sqrt{F(x,Ty)+F(y,Tx)}]}{[1+\sqrt{F(x,Tx)+F(y,Ty)}]} \quad (1)$$

where α , β and $\gamma \ge 0$ are constant such that $\alpha+\beta+\gamma < 1$. If for some x_0 ε X, the sequence of iterates $\{T^n{}_k x_0\}$ covering to z ε X then z is a fixed point of T.

Proof: We have the monotonic sequence of noon negative real numbers.

$$F(x,T x_0) > f(Tx, T^2 x_0) >------> f(T^n x_0, T^{n+1} x_0) -----------$$

which must converge along with all its subsequences to some real number λ .

Now from the continuity of F and T we have

$$
\begin{aligned}
F(z,Tz) &= F\left(\lim_{k\to\infty} T^{n_k} x_0,\ T\lim_{k\to\infty} T^{n_k} x_0\right)\\
&= F\left(\lim_{k\to\infty} T^{n_k} x_0,\ \lim_{k\to\infty} T^{n_k+1} x_0\right)\\
&= \lim_{k\to\infty} F\left(T^{n_k} x_0,\ T^{n_k+1} x_0\right)\\
&= \lim_{k\to\infty} F\left(T^{n_k+1} x_0,\ T^{n_k+2} x_0\right)\\
&= F\left(\lim_{k\to\infty} T^{n_k+1} x_0,\ \lim_{k\to\infty} T^{n_k+2} x_0\right)\\
&= F(Tz, T^2z)
\end{aligned}
$$

If $z \neq Tz$ then form (1) we have,

$$
\begin{aligned}
F(Tz,T^2z) \le\ & \alpha \frac{F(z,Tz)F(Tz,T^2z)}{F(z,T^2z)+F(Tz,Tz)+F(z,Tz)}\\
&+\beta \frac{F(z,Tz)\left[1+\sqrt{F(z,Tz)F(z,Tz)}+\sqrt{F(z,Tz)F(Tz,Tz)}\right]}{[1+F(z,Tz)+F(z,Tz)F(z,T^2z)F(Tz,Tz)F(Tz,T^2z)]}\\
&+\gamma \frac{F(Tz,T^2z)\,]\left[1+\sqrt{F(z,T^2z)+F(Tz,Tz)}\right]}{\left[1+\sqrt{F(z,Tz)+F(Tz,T^2z)}\right]}\\
\le\ & \alpha \frac{F(z,Tz)F(Tz,T^2z)}{F(Tz,T^2z)}\\
&+\beta \frac{F(z,Tz)[1+F(z,Tz)]}{[1+F(z,Tz)]}\\
&+\gamma \frac{F(Tz,T^2z)\left[1+\sqrt{F(z,T^2z)}\right]}{\left[1+\sqrt{F(Tz,T^2z)}\right]}\\
=\ & \alpha F(z,Tz)+\beta F(z,Tz)+\gamma F(Tz,T^2z)
\end{aligned}
$$

$$(1-\gamma)\ F(Tz,T^2z) \le (\alpha+\beta)\ F(z,Tz)$$

$$F(Tz,T^2z) \le \left(\frac{\alpha+\beta}{1-\gamma}\right) F(z,Tz)$$

which gives ,

$$F(z,Tz) = F(Tz,T^2z) < F(z,Tz)$$

where $\alpha+\beta+\gamma < 1$. Which is contradiction.

Thus z is α fixed point of F.

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