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Mathematical Models for Stability in Computer Network

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ABSTRACT

To protect cyber world from different kind of malicious objects, an attempt has been made to develop an e-epidemic SIQRS (Susceptible, Infectious, Quarantine, Recovered Susceptible) model for the transmission of malicious objects in computer network. Basic reproduction number and Stability of the system has been discussed by Jacobian Method. Differential equations are employed to simulate the system which may help us to understand the attacking behavior of the malicious objects in computer network.

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1. INTRODUCTION

As a technical term coined by Cohen, a computer virus is a malicious program that can replicate itself and spread from computer to computer. Once breaking out, a virus can perform devastating operations such as modifying data, deleting data, deleting files, encrypting files, and formatting disks. The viruses are being developed simultaneously with the computer systems and the use of INTERNET facilities increases the number of damaging virus incidents. Virus program must be executed in order to infect a computer system. Viruses can attach themselves to other programs in order to ensure that this happens. It is a program which can destroy or cause damage to data stored on a computer system . Controlling the malicious objects in computer network has been an increasingly complex issue in recent years. In order to curb the malicious object and analyze the stability of the system, we propose an SIQRS model. In this model, we have used Quarantine class for the very general form of nonlinear incidence rate as a device. Quarantine the word symbolizing a force isolation.

Anderson and May[9,10] discussed the spreading nature of biological viruses, parasite etc leading to infectious diseases in human population through several epidemic models. The action of malicious objects throughout a network can be studied by using epidemiological models for disease propagation[1,6]. Richard and Mark[2,11] proposed an improved SEI (Susceptible-Exposed-Infected) model to simulate virus propagation. However, they do not show the length of latency and take into account the impact of anti-virus software. Mishra and Saini[3] presented a SEIRS model with latent and temporary immune periods to overcome limitation, which can reveal common worm propagation. Wa and Feng[8] showed that an epidemic approximation near threshold number can have a homoclinic bifurcation, so that some perturbation of the original model might also have a homoclinic bifurcation. Several authors studied the global stability of several epidemiological models[4,5,7].

Several authors have studied on bilinear standard incidence rate, but these may require modification. In this work, we assume a general form f(S, I, N) as a non-linear incidence rate constrained with a few e-epidemic feasible conditions. We show that for SIQRS models

(a) if basic reproduction number, that is, $R_0 > 1$ then the endemic equilibrium of the system asymptotically stable, and
(b) if $R_0 \leq 1$, then there is no endemic equilibrium state, and the worm infection-free equilibrium state is asymptotically stable.

In the next stage we again show that for the instability of the endemic equilibrium state, the incidence rate of f(S, I, N) must be convex with respect to infection I. We postulate that the incidence rate depends on the variables S, I, and N only and is given by a function f(S, I, N).

The function f(S, I, N) must satisfy the conditions

$$\left.\begin{aligned} &\mathrm{f}(\mathrm{S},0,\mathrm{N}) = \mathrm{f}(0,\mathrm{I},\mathrm{N}) = 0 \\ &\text{and } \frac{\partial f(S,I,N)}{\partial I} > 0, \frac{\partial f(S,I,N)}{\partial S} > 0, \text{for all } \mathrm{S},\mathrm{I} > 0 \end{aligned}\right\} \qquad (1)$$

We also assume that the function f(S,I,N) is concave with respect to the variable I;

$$\text{i.e,} \qquad \frac{\partial^2 f(S,I,N)}{\partial I^2} \leq 0 \quad \text{for all S,I} > 0 \qquad (2)$$

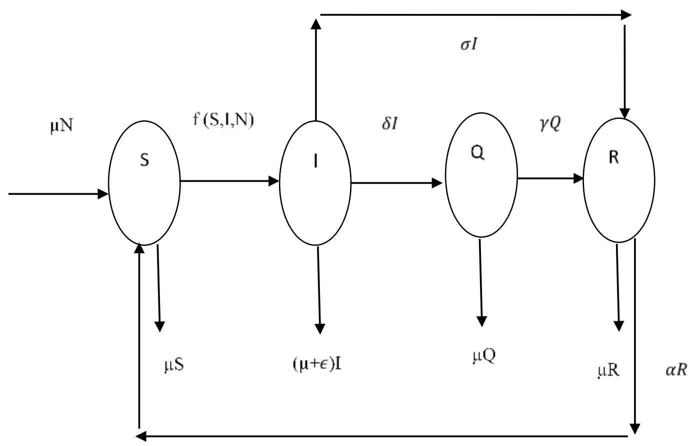

Figure 1: Schematic diagram for flow of malicious objects in computer network

We proposed the model represented in Fig. 1 for the dynamics of the infection propagation in a computer network. The model contains a modification related to the traditional SIR model[17]. In this model, we have used Quarantine class for the very general form of nonlinear incidence rate as a device. Quarantine the word symbolizing a force isolation. The biological world witnessed a radical changes in the last decades with successful implementation of Quarantine to reduce the transmission of human disease such as Leprosy, Plague, Smallpox etc. The same concept has been adopted in the cyber world; the most infected nodes are isolated from the computer network till they get recovered. In the SIQRS model for the infections that does not confer permanent immunity

The total population N is divided into four groups: susceptible nodes go to infectious class, thereafter some nodes remain in the infected class while they are infectious and then move to the recovered class after the run of anti malicious software and other most infected nodes are transferred into the quarantined class while they are infectious and then move to the recovered class after their recovery. The model here have a variable total population size, because they have recruitment into the susceptible class by inclusion of some new nodes and they have crashing of nodes due to the reason other than the attack of malicious Objects.

2. MATHEMATICAL FORMULATION FOR THE SIQR MODEL WITH NON LINEAR INCIDENCE RATE

$$\left.\begin{aligned} \frac{dS}{dt} &- \mu N - f(S,I,N) - \mu S + \alpha R \\ \frac{dI}{dt} &= f(S,I,N) - (\delta + \mu + \epsilon + \sigma)I \\ \frac{dQ}{dt} &= \delta I - (\gamma + \mu)Q \\ \frac{dR}{dt} &= \sigma I + \gamma Q - (\mu + \alpha)R \end{aligned}\right\} \tag{3}$$

Where N(t): the total population size in time t; S(t): the number of susceptible at time t; I(t) : the number of infected nodes; Q(t): the number of quarantined nodes; R(t): the recovered nodes after the run of anti malicious software; μ: the birth rate and death rate due to the reason other than the attack of malicious objects; ϵ : the death rate in infective compartment due to malicious objects; δ : the rate constant which leaves the infective class for quarantine class; γ: the rate by which the nodes go from quarantine class into recovered class; α : the rate of transmission of nodes from recovered to susceptible class; σ : the rate of transmission of nodes from infectious to recovered class.

We have N = S + I + Q + R

Thus, the reduced system of equations takes form,

$$\left.\begin{aligned} \frac{dS}{dt} - \mu + \alpha)N - f(S,I,N) - (\mu+\alpha)S - \alpha(I+Q) \\ \frac{dI}{dt} = f(S,I,N) - (\delta+\mu+\epsilon+\sigma)I \\ \frac{dQ}{dt} = \delta I - (\gamma+\mu)Q \end{aligned}\right\} \quad (4)$$

At equilibrium state

$$\left.\begin{aligned} (\mu+\alpha)N = f(S,I,N) + (\mu+\alpha)S + \alpha(I+Q) \\ f(S,I,N) = (\delta+\mu+\in+\sigma)I \\ \delta I = (\gamma+\mu)Q \end{aligned}\right\} \quad (5)$$

At worm infection free equilibrium state, $G_0 = (S_0, I_0)$, where $S_0 = N$ and $I_0 = 0$. Apart from the infection free equilibrium state G_0, the system can have positive endemic equilibrium state G*, then we have following lemma:

Lemma 1-: If the condition f(S, 0, N) = 0 = f (0, I, N) and $\frac{\partial f(S,I,N)}{\partial I} > 0$, $\frac{\partial f(S,I,N)}{\partial S} > 0$for all S, I > 0; $\frac{\partial^2}{\partial I^2} f(S,I,N) \leq 0$ for all S, I > 0 then at the endemic equilibrium state $I \in (0, I^*)\, \frac{\partial f(S^*,I^*,N)}{\partial I} \leq (\delta+\mu+\epsilon+\sigma)$ where the strict equality hold only if $\frac{\partial^2 f(S^*,I,N)}{\partial I^2} = 0$ for all $I \in (0, I^*)$

Proof: - From equation (5) at endemic equilibrium state, $f(S^*, I^*, N) = (\delta+\mu+\epsilon+\sigma)I^*$

Let, $\bar{f}(I) = f(S^*, I, N)$.

Also assume that $\frac{\partial f(S^*,I^*,N)}{\partial I} = \frac{d\bar{f}(I^*)}{dI} > (\delta+\mu+\epsilon+\sigma)$ (6)

By the mean value theorem, if $I_1 \in (0, I^*)$ then

$$\frac{d\bar{f}(I_1)}{dI} = \frac{\bar{f}(I^*) - \bar{f}(0)}{I^* - 0} = \frac{f(S^*,I^*,N) - f(S^*,0,N)}{I^*} = \frac{f(S^*,I^*,N) - 0}{I^*} = \frac{f(S^*,I^*,N)}{I^*} = \frac{(\delta+\mu+\epsilon+\sigma)I^*}{I^*} = (\delta+\mu+\epsilon+\sigma)$$

If $I_0 \in (I_1, I^*)$ then again using the mean value theorem

$$\frac{\partial^2 f(S^*,I_0,N)}{\partial I^2} = \frac{\partial^2 \bar{f}(I_0)}{\partial I^2} = \frac{\frac{d\bar{f}(I^*)}{dI} - \frac{d\bar{f}(I_1)}{dI}}{I^* - I_1} = \frac{\frac{d\bar{f}(I^*)}{dI} - (\delta+\mu+\epsilon+\sigma)}{I^* - I_1} > 0$$

$$\Rightarrow \frac{\partial^2 f(S^*,I_0,N)}{\partial I^2} > 0$$

which contradicts the hypothesis of the Lemma and hence

$\frac{\partial f(S^*,I^*,N)}{\partial I} \leq (\delta + \mu + \epsilon + \sigma)$

We define the basic reproduction number of the system, if $\frac{dI}{dt} > 0$ then malicious object will spread in the system i.e. $f(S,I,N) - (\delta + \mu + \epsilon + \sigma) > 0$

$f(S,I,N) > (\delta + \mu + \epsilon + \sigma)$

$\Rightarrow \frac{1}{(\delta+\mu+\epsilon+\sigma)} \times \frac{\partial f(S,I,N)}{\partial I} > 1$ then infection will spread in the system and it is denoted by R_0 and is called basic reproduction number.

Therefore $R_0 = \frac{1}{(\delta+\mu+\epsilon+\sigma)} \times \frac{\partial f(S,I,N)}{\partial I}$

Theorem 1:(a) If the incidence rate f(S, I, N) satisfies the conditions (1) and (2) and if $R_0 > 1$, then the endemic equilibrium state G* = (S*, I*) of the system (4) is asymptotically stable.

(b) If $R_0 \leq 1$, then there is no endemic equilibrium state, and the worm infection – free equilibrium state is asymptotically stable.

Proof (a): Jacobian of the system (4) is

$$J = \begin{bmatrix} -\frac{\partial f(S,I,N)}{\partial S} - (\mu+\alpha) & -\frac{\partial f}{\partial I} - \alpha & -\alpha \\ \frac{\partial f(S,I,N)}{\partial S} & \frac{\partial f}{\partial I} - (\delta + \mu + \in + \sigma) & 0 \\ 0 & \delta & -(\gamma+\mu) \end{bmatrix}$$

By the Routh-Hurwitz Criterion, the eigen values of the matrix have negative real parts if and only if the inequalities $a_1, a_2, a_3 > 0$ and $a_1a_2 - a_3 > 0$, hold for the coefficient of the characteristic equation $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$

Characteristic equation of the above matrix

$$|J - I\lambda| = \begin{vmatrix} -\frac{\partial f(S,I,N)}{\partial S} - (\mu+\alpha) - \lambda & -\frac{\partial f}{\partial I} - \alpha & -\alpha \\ \frac{\partial f(S,I,N)}{\partial S} & \frac{\partial f}{\partial I} - (\delta + \mu + \in + \sigma) - \lambda & 0 \\ 0 & \delta & -(\gamma+\mu) - \lambda \end{vmatrix} = 0$$

$$-(\frac{\partial f}{\partial S} + \mu + \alpha + \lambda)\left\{-(\gamma + \mu + \lambda)(\frac{\partial f}{\partial I} - (\delta + \mu + \in + \sigma + \lambda)\right\} - (\frac{\partial f}{\partial I} + \alpha)\frac{\partial f}{\partial S}(\gamma + \mu + \lambda) - \alpha\frac{\partial f}{\partial S}\delta = 0$$

$$\Rightarrow -(\frac{\partial f}{\partial S} + \mu + \alpha + \lambda)\left\{\gamma\delta + \mu\gamma + \in\gamma + \sigma\gamma + \lambda\gamma - \gamma\frac{\partial f}{\partial I} + \mu\delta + \mu^2 + \mu\in + \sigma\mu + \lambda\mu - \mu\frac{\partial f}{\partial I} + \lambda\delta + \lambda\mu + \lambda\in + \lambda\sigma + \lambda^2 - \lambda\frac{\partial f}{\partial I}\right\} - (\frac{\partial f}{\partial I} + \alpha)\frac{\partial f}{\partial S}(\gamma + \mu) - \lambda(\frac{\partial f}{\partial I} + \alpha)\frac{\partial f}{\partial S} - \alpha\frac{\partial f}{\partial S}\delta = 0$$

The above characteristic equation is identical with

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

Coefficient of $\lambda^2 = a_1 = \frac{\partial f}{\partial S} + 2\mu + \alpha + \gamma > 0$

Coefficient of $\lambda = a_2 =$

$$\left(\gamma\delta + \mu\gamma + \in\gamma + \sigma\gamma - \gamma\frac{\partial f}{\partial I} + \delta\mu + \mu^2 + \in\mu + \sigma\mu - \mu\frac{\partial f}{\partial I}\right) + \left(\frac{\partial f}{\partial S} + \mu + \alpha\right)\ \gamma + \mu\ -$$

$$\frac{\partial f}{\partial S}\left(\frac{\partial f}{\partial I} + \alpha\right)$$

Coefficient of $\lambda^0 = a_3 =$

$$\left(\frac{\partial f}{\partial S} + \mu + \alpha\right)\left(\gamma\delta + \mu\gamma + \in\gamma + \sigma\gamma - \gamma\frac{\partial f}{\partial I} + \mu\delta + \mu^2 + \mu\in + \sigma\mu - \mu\frac{\partial f}{\partial I}\right) + \left(\frac{\partial f}{\partial I} + \alpha\right)\frac{\partial f}{\partial S}\ \gamma + \mu$$

$$+\alpha\frac{\partial f}{\partial S}\delta$$

We obtain, $a_1a_2 - a_3 > 0$.

Therefore, all three roots of the characteristic equation have negative real parts, and hence the endemic equilibrium state G^* is asymptotically stable.

Proof (b):- Since the diagonal elements are real and negative of the above matrix therefore the system is asymptotically stable for the malicious objects free equilibrium state.

Parametric values used in simulation μ=0.05, β =0.001, δ =0.005, ϵ=0.26, σ =0.992, γ =0.008, α =0.007

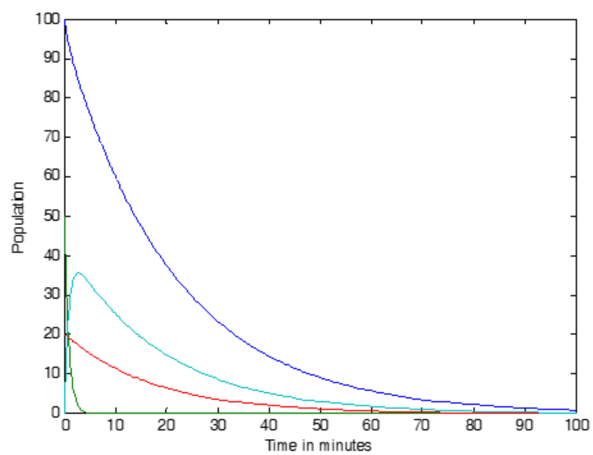

Figure 2: Dynamic behavior of system (3)

3. CONCLUSION

According to biological e-epidemic model, we develop SIQRS for the transmission of malicious objects with non-linear incidence rate in computer network. With the help of jacobian matrix, there is no endemic equilibrium state and the worm infection-free equilibrium state is asymptotically stable, if $R_0 \leq 1$ and the endemic equilibrium of the system asymptotically stable, if $R_0 > 1$. If we take βSI instead of incidence rate f(S,I,N) and simulate the system (3); we get the above fig (2), fig (3), fig (4). With the help of these graphs software company may develop power full antivirus software, which may protect the cyber world.

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