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Arithmetic - Geometric Indices of Path Graph

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ABSTRACT

Let G be a molecular graph. The Arithmetic-Geometric (AG) indices of G are defined as AG (G)=$\sum_{uv\in E(G)}\frac{du+dv}{2\sqrt{du*dv}}$ where du(or dv) denote the degree of the vertex u(or v), respectively. In this paper the AG indices of path graph are obtained.

Keywords: Chemical graph, Molecular graph, Arithmetic- Geometric index.

1. INTRODUCTION

A graph G is a pair G=(V,E) consisting of a finite set V and set E of 2-element subsets of V (infinite graphs are also studied, here we consider only finite graphs).The elements of V are called vertices (points, nodes, junctions, or 0-simplexes) and elements of E are called edges (lines, arcs, branches or 1-simplexes).The set V is known as the vertex set of G and E as the edge set of G. If we denote the edge by e, we can then write e=uv which is an edge of G. Thus two vertices u and v of G are said to be adjacent, if an edge e joins u and v, and two edges are adjacent if they have a common vertex.

Chemical graph theory is branch of mathematical chemistry which deals with the nontrivial applications of graph theory to solve molecular problem. In general, a graph is used to represent a molecule by considering the atoms as the vertices of the graph and the molecular bonds as the edges.

The degree of a vertex v of G is the number of edges incident with v and is written deg(v).

MOLECULAR GRAPH

A molecular graph G= (V, E) is a simple graph having n=|V| nodes and m=|E| edges. The nodes $v_i \in V$ represent non-hydrogen atoms and the edges $(v_i , v_j)\in E$ represent covalent bonds between the corresponding atoms. In particular, hydrocarbons are formed only by carbon and hydrogen atom and their molecular graphs represent the carbon skeleton of the molecule. Consider the alkane molecule Propane,

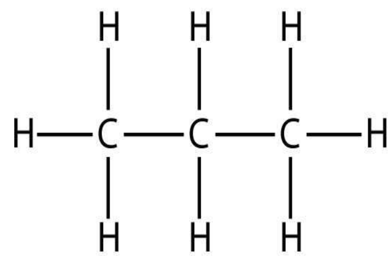
Alkane molecule

↓

C_3H_8 (Propane)

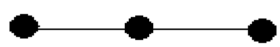
Chemical formula

↓



Structural formula

↓



Molecular hydrogen-suppressed graph

In this paper we consider simple connected graph with no self-loops and no multiple edges.

The atom-bond connectivity index (ABC index) was first proposed by Ernesto Estrada in 1998 and is defined as follows

$$ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{du+dv-2}{du*dv}}$$

In 1975, Randic introduced the connectivity index, namely Randic index(G) to reflect molecular branching , which is defined as

$$(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{du*dv}}$$

Vukicevic and Furtula (2009) proposed a topological index named geometric-arithmetic index denoted by GA and is defined by

GA(G)=$\sum_{uv\in E(G)}\frac{2\sqrt{du*dv}}{du+dv}$

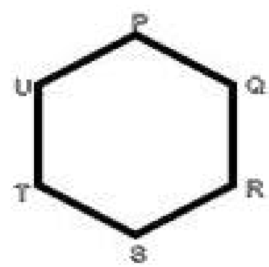
We recall the following topological indices to find out new index named asarithmetic-geometric index denoted as AG index and is defined by

AG(G)=$\sum_{uv\in E(G)}\frac{du+dv}{2\sqrt{du*dv}}$

Where, AG index is considered for distinct vertices.
The above equation is the sum of the ratio of the Arithmetic mean and Geometric mean of u and v.where du(or dv) denote the degree of the vertex u(or v).

Example:

Let G be the simple graph on six vertices namely P, Q, R, S, T, U. Then



AG(G)=$\sum_{uv\in E(G)}\frac{du+dv}{2\sqrt{du*dv}}$

$$=\frac{dP+dQ}{2\sqrt{dP*dQ}}+\frac{dP+dR}{2\sqrt{dP*dR}}+\frac{dP+dS}{2\sqrt{dP*dS}}+\frac{dP+dT}{2\sqrt{dP*dT}}+\frac{dP+dU}{2\sqrt{dP*dU}}$$

$$=\frac{2+2}{2\sqrt{2*2}}+\frac{2+2}{2\sqrt{2*2}}+\frac{2+2}{2\sqrt{2*2}}+\frac{2+2}{2\sqrt{2*2}}+\frac{2+2}{2\sqrt{2*2}}$$

$$=\frac{4}{2\sqrt{4}}+\frac{4}{2\sqrt{4}}+\frac{4}{2\sqrt{4}}+\frac{4}{2\sqrt{4}}+\frac{4}{2\sqrt{4}}$$

$$=\frac{4}{2*2}+\frac{4}{2*2}+\frac{4}{2*2}+\frac{4}{2*2}+\frac{4}{2*2}$$

= 1+1+1+1+1
= 5

MAIN RESULTS

Theorem:
The AG index of path graph of order ‘n’ is given by

$$AG(G) = \begin{cases} \frac{3}{\sqrt{2}}\sum_{n=0}^{m} n + k, \text{ for even path} \\ \left[\frac{3}{\sqrt{2}}\sum_{n=1}^{m} n - \frac{3n}{2\sqrt{2}}\right] + k, \text{ for odd path} \end{cases}$$

Proof:

The AG index is

$$AG(G)=\sum_{uv\in E(G)} \frac{du+dv}{2\sqrt{du*dv}}$$

Case-1: For even path

If P_n denotes an even path of order n then

$AG(P_2) = 1$

$AG(P_4) = \frac{3}{\sqrt{2}} + 1$

$AG(P_6) = \frac{6}{\sqrt{2}} + 1$

.

.

.

$AG(P_m) = \frac{3m}{\sqrt{2}} + 1$

Adding all the above equations:

$$AG(P_2)+AG(P_4)+AG(P_6)+\ldots\ldots+AG(P_m)= 1+\left(\frac{3}{\sqrt{2}}+1\right)+\left(\frac{6}{\sqrt{2}}+1\right)+\ldots\ldots+\left(\frac{3m}{\sqrt{2}}+1\right)$$

$$\sum_{n=2}^{m} AG(P_n) = 1+\left(\frac{3}{\sqrt{2}}+1\right)+\left(\frac{6}{\sqrt{2}}+1\right)+\ldots\ldots+\left(\frac{3m}{\sqrt{2}}+1\right)$$

$$=\left(\frac{3(0)}{\sqrt{2}}+1\right)+\left(\frac{3(1)}{\sqrt{2}}+1\right)+\left(\frac{3(2)}{\sqrt{2}}+1\right)+\ldots\ldots+\left(\frac{3(m)}{\sqrt{2}}+1\right)$$

$$=\left[\frac{3(0)}{\sqrt{2}}+\frac{3(1)}{\sqrt{2}}+\frac{3(2)}{\sqrt{2}}+\ldots\ldots\ldots\ldots+\frac{3(m)}{\sqrt{2}}\right]+\left[\underbrace{1+1+1+1+\ldots\ldots\ldots\ldots+1}_{\text{k}-\text{times}}\right]$$

$$=\frac{3}{\sqrt{2}}\left[0+1+2+3+\ldots\ldots\ldots+m\right]+\left[\underbrace{1+1+1+1+\ldots\ldots\ldots\ldots+1}_{\text{k}-\text{times}}\right]$$

$$=\frac{3}{\sqrt{2}}\left[1+2+3+\ldots\ldots\ldots+m\right]+\left[\underbrace{1+1+1+1+\ldots\ldots\ldots\ldots+1}_{\text{k}-\text{times}}\right]$$

$$=\frac{3}{\sqrt{2}}\sum_{n=0}^{m} n+\sum 1$$

$$=\frac{3}{\sqrt{2}}\sum_{n=0}^{m} n + \text{k. Where n=0,1,2,3,…..,m}$$

Case-2: For odd Path

If P_n denotes an odd path of order n then

$AG(P_1) = 0$

$AG(P_3) = \frac{3}{2\sqrt{2}} + 1$

$AG(P_5) = \frac{9}{2\sqrt{2}} + 1$

.

.

.

$AG(P_m) = \frac{3m}{2\sqrt{2}} + 1$

Adding all the above equations:

$$AG(P_1)+AG(P_3)+AG(P_5)+\ldots\ldots\ldots\ldots+AG(P_m) = 0+\left(\frac{3}{2\sqrt{2}}+1\right)+\left(\frac{9}{2\sqrt{2}}+1\right)+\ldots\ldots\ldots+\left(\frac{3m}{2\sqrt{2}}+1\right)$$

$$\sum_{n=1}^{m} AG(P_n) = 0+\left(\frac{3}{2\sqrt{2}}+1\right)+\left(\frac{9}{2\sqrt{2}}+1\right)+\ldots\ldots\ldots\ldots+\left(\frac{3m}{2\sqrt{2}}+1\right)$$

$$=\left(\frac{3(1)}{2\sqrt{2}}+1\right)+\left(\frac{3(3)}{2\sqrt{2}}+1\right)+\left(\frac{3(5)}{2\sqrt{2}}+1\right)+\ \ldots\ldots\ldots\ldots\ldots\ldots+\left(\frac{3(\mathrm{m})}{2\sqrt{2}}+1\right)$$

$$=\left[\frac{3(1)}{2\sqrt{2}}+\frac{3(3)}{2\sqrt{2}}+\frac{3(5)}{2\sqrt{2}}+\ldots\ldots\ldots\ldots+\frac{3(m)}{2\sqrt{2}}\right]+\left[\underbrace{1+1+1+1+\ldots\ldots\ldots\ldots+1}_{\mathrm{k-times}}\right]$$

$$=\frac{3}{2\sqrt{2}}\left[1+3+5+\ldots\ldots\ldots+m\right]+\left[\underbrace{1+1+1+1+\ldots\ldots\ldots\ldots+1}_{\mathrm{k-times}}\right]$$

$$=\frac{3}{2\sqrt{2}}\sum_{n=1}^{m}(2n-1)+\sum 1$$

$$=\left[\frac{3}{2\sqrt{2}}\sum_{n=1}^{m}2n-\frac{3}{2\sqrt{2}}\sum_{n=1}^{m}1\right]+\mathrm{k}$$

$$=\left[\frac{3}{\sqrt{2}}\sum_{n=1}^{m}n-\frac{3n}{2\sqrt{2}}\right]+\mathrm{k}.\ \text{Where n=1,3,5,.....,m}$$

Open problems for further indices: So far we have results on path graphs, and other class of graphs are under investigation.

REFERENCES

1. K. C. Das, Atom-bond connectivity index of graphs, *Discr. Appl. Math*.158, 1181-1188 (2010).
2. K.C. Das, I. Gutman, B. Furtula, On atom-bond connectivity index, *Chem. Phys. Lett*. 511, 452-454 (2011).
3. E. Estrada, Atom-bond connectivity and the energetic of branched alkanes, *Chem. Phys. Lett.* 463, 422-425 (2008).
4. B. Furtula, A. Graovac, D. Vukicevic, Atom-bond connectivity index of tree, *Discrete Appl. Math.*157, 2828-2835 (2009).
5. K.C. Das, N. Trinajstic, Comparison between first geometric-arithmetic index and atom-bond connectivity index, *Chem. Phys. Lett*. 497, 149-151 (2010).
6. Xing, R., Zhou. B , and Zhibin D. Further results on atom-bond connectivity index of trees. *Discrete Applied Mathematics*, 158, 1536-1545 (2009).
7. Trinajstic, N: Chemical Graph Theory.CRC Press, Boca Raton(1992).
8. D. Vukicevic and B. Furtula, Topological index based on the ratios of geometrical and arithmetical mean of end-vertex degrees of edges, *J.Math.Chem*.,26, 1369-1376 (2009).