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An Alternative Approach of Project Scheduling Using Fuzzy Logic and Probability Distribution

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ABSTRACT

Project Management concerns the scheduling and control of activities in such a way that the project can be completed within the minimum time and budget. In real situation, operations times of activities in a project network are difficult to define and estimate exactly. However, it is very important to make most optimistic time more reliable and accurate. This paper presents a new effective algorithmic approach for making most optimistic time more accurate than the conventional method, by using Fuzzy activity times and probability distribution. Fuzzy α cuts and Monte Carlo Simulation are used to the operation times for making accurate estimate of each activity time and, an advanced FPERT method is used to identify possible Critical Path of the Project . In addition to, a new technique has to be established for checking the criticality of the activity and Critical Path. Finally, an example is to present for verifying proposed FPERT method.

Keywords: Fuzzy activity, Fuzzy α cuts, Monte Carlo Simulation, Criticality of activity.

1. INTRODUCTION

To ensure the project's success, the project manager must identify the actual and more accurate schedule, though it is very tough in reality; so he has to maintain a network to complete each and every task in the project. This network contains a set of activities that must be performed according to precedence constraints starting which activities must start after the completion of specified other activities. Such a project network can be represented as a direct graph, but usually they use FPERT Network method where the activity-on-arrow (AOA) graph and the activity-on-node (AON) graph[1]. It is hard to get precise information about activity durations in some situation, such as early rough planning in long range projects. Therefore, it is important to compute three time estimates efficiently. Basically, Most Optimistic time, it plays vital rule for total duration of the project and, then the selection of Critical Path of activity. So in this paper, authors find a scope to deal with most optimistic time and degree of Criticality of Critical Path.

When the activity times in the project are deterministic and known, critical path method has been demonstrated to be a useful tool in managing a projects in an efficient manner to meet this challenge[2], also PERT technique becomes a useful technique when there is imprecise data[3,4] and Monte Carlo simulation[5,6] based on the probability theory can be employed. In PERT analysis, the use of beta distribution or its variants may not be able to provide an appropriate distribution when the activity times are highly skewed[7]. An alternative way to deal with imprecise data is to employ the concept of fuzziness[8], whereby the vague activity times can be represented by fuzzy sets. There are so many techniques where activity times in a project are approximately known and are more suitably represented by fuzzy sets rather than crisp number[9,10]. In particular, the problems of computing the intervals of possible values of the latest starting times and floats of activities with imprecise durations represented by fuzzy number. Most of them are based on the deterministic CPM with formulas for the forward and backward recursions, in which the deterministic activities times are replaced with the fuzzy activity times. But, as noted by Zieliniski[10], backward recursion fails to compute the sets of possible values of the latest starting times and floats of activities. Moreover, for the same path, different definitions of the fuzzy critical path give different estimations of the degree of criticality. Debois *et. al*., proposed several heuristics for computing the sets of possible values of the latest starting times and float of activities. Zielinski[10] developed new polynomial algorithms for determining the intervals of the latest starting times in the general network. Chanas and Zielinski[11] discussed the complexity of criticality. Chanas and Zielinski[6] proposed a natural generalization of the criticality concept for project networks with interval and fuzzy activity times, in which two methods of calculating the degree of possible criticality and some results are provided. Chanas[5] proposed approach can describes the total duration time of project network via a membership function which completely conserves all the fuzziness of activity times, and the critical paths under different possibility levels can also be obtained. Clearly, when activity times in a project network are fuzzy, the total duration time needed to complete the project will be fuzzy also. Our proposed method, is able to calculate fuzzy activity time for each activity more accurately than conventional method, also the fuzzy total duration of project Network. Though K. Achrysafis and B.K Papadopoulos[7] explains fuzzy field generalized beta distribution and Critically test, but the reason is unexplained. However, there are still several unsolved issues in Fuzzy PERT method.

2. BACKGROUND

2.1 Fuzzy concepts

A fuzzy sets can be mathematically constructed by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set[10,11]. This grade corresponds to the individual's similarity to the concept represented by fuzzy set. These membership grades are often represented by real numbers ranging from a minimum of 0 to maximum of 1[3,14].

The fuzzy number $\tilde{A}$ is a fuzzy set whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following[14]:

1. $\mu_{\tilde{A}}(x)$ a piecewise continuous
2. $\mu_{\tilde{A}}(x)$ is a convex fuzzy subset;
3. $\mu_{\tilde{A}}(x)$ is the normality of a fuzzy subset, implies that for at least one element x_0 the membership grade must be 1;i.e., $\mu_{\tilde{A}}(x_0) = 1$

2.2 Triangular Fuzzy number in FPERT

Particularly common in practical applications are triangular fuzzy numbers, denoted $\tilde{T} = (O,\ M_0,\ P)$ when $O > 0$, $\tilde{T}$ is a positive triangular fuzzy number (PTFN) [5, 15]. The membership function of positive triangular fuzzy number $\tilde{T}$ defined as:

$$\mu_{\tilde{T}}(x) = \begin{cases} \dfrac{x-O}{M_o - O} & ,\ O \le x \le M_o \\ \dfrac{P-x}{P-M_o} & ,\ M_o \le x \le P \\ 0 & ,\ otherwise \end{cases} \quad \text{where, } O > 0\ .$$

2.3 Fuzzy Ranking Method

Ranking fuzzy numbers applied in fuzzy PERT is used to determine the earliest starting time. Lee and Li[6] presented the generalized mean value method, an effective method for ranking and comparing fuzzy numbers. For a triangular fuzzy numbers $\tilde{T} = (O,\ M_0,\ P)$, the generalized mean value $G(\tilde{T})$ and Standard deviation $S(\tilde{T})$ are given by:

$$G(\tilde{T}) = \frac{O + M_o + P}{3}\text{, and } S(\tilde{T}) = \frac{1}{18}[O^2 + M_o{}^2 + P^2 - O.M_o - O..P - M_o.P]$$

2.4 Fuzzy α cuts

Approach to construct the membership functions if $\mu_{\tilde{D}}(d)$ is to derive the α cuts of $\mu_{\tilde{D}}(d)$. The α cut of $\tilde{T}_{ij}$ is as follows:

$$(\tilde{T}_{ij})_{\alpha} = \{ t_{ij} \in S(\ : \mu_{\tilde{A}}(\tilde{T}_{ij}) \ \ | \mu_{\tilde{T}_{ij}}(\tilde{T}_{ij}) \geq \alpha\}$$

Here, $(\tilde{T}_{ij})$ is a crisp set rather than a fuzzy set. Using α - cuts, $\tilde{T}_{ij}$ can be represented by different levels of confidence intervals[17]. Besides these Klir and Yuan[14] present the fundamentals of fuzzy arithmetic, which is based on two properties of fuzzy numbers:

Property 1: Each fuzzy set and thus also each fuzzy number, can fully and uniquely be represent by its α cuts.

Property 2: The α cuts of each of fuzzy number are closed intervals of real number for all $\alpha \in (0, 1]$

These properties enable to define arithmetic operations on fuzzy numbers, in terms of arithmetic operations, on their α cuts[7].

2.5 Distribution and statistics On the Fuzzy activity times

As we stated earlier, three times estimates are very difficult to prepare, unless some guidance are available. The planar should base the estimations on available information and past experience. Here the case is uncertain; therefore, it is crucial need to know their time distributions.

2.6 Operation time distribution

Monte Carlo Simulation which generates some pseudo points in between optimistic and pessimistic time for plotting the curve between the 'time 'of completion and the number of jobs completed in that 'time'. These curves will be any one of the following:

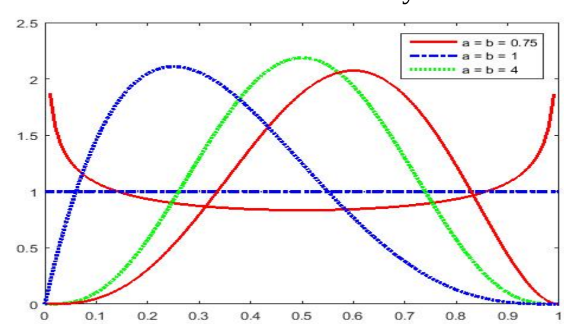

Figure 1 Time distribution

These curves are the beta distribution curve. The beta distribution is characterized by the density function

$$f_x(x) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\,\Gamma(\beta)}\, x^{\alpha}(1-x)^{\beta-1}, & for \quad 0 \le x \le 1 \\ 0, & otherwise \end{cases} \tag{1}$$

where, parameter α and β take only positive values. The coefficient of $f_x(x)$, $\dfrac{\Gamma(\alpha+\beta)}{\Gamma\alpha.\,\Gamma\beta}$ can be represented by $\dfrac{1}{B(\alpha,\beta)}$, where $\dfrac{1}{B(\alpha,\beta)} = \dfrac{\Gamma(\alpha+\beta)}{\Gamma\alpha.\,\Gamma\beta}$ is known as the beta function. The parameters α and β are both shape parameters. Finally, as a special case, the uniform distribution over interval $(0,1)$ results when $\alpha = \beta = 1$(Fig. shows in above)

Because of its versatility over a finite interval, the beta distribution is used to represent a large number of physical quantities for which values are restricted to an identifiable interval. Suppose a random phenomenon Y can be observed independently n times and, after these n independent observations are ranked in order of increasing magnitude, let y_r and y_{n-s+1}, be the values of the rth smallest and sth largest observation respectively. If random variable X is used to denote the proportion of the original Y taking values between y_r and y_{n-s+1}, it can be shown that X follows a beta distribution with $\alpha = n-r-s+1$, and $\beta = r+s$; that is,

$$f_x(x) = \begin{cases} \dfrac{\Gamma(n+1)}{\Gamma(n-r-s+1).\,\Gamma(r+s)}\, x^{n-r-s}(1-x)^{r+s-1}, & for \quad 0 \le x \le 1 \\ 0, & otherwise \end{cases} \tag{2}$$

This result can be found in Wilks (1942)[19].

2.7 Probability Tabulations

The probability distribution function associated with the beta distribution is:

$$F_x(x) = \begin{cases} 0, & for \quad x < 0; \\ \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\,\Gamma(\beta)} \displaystyle\int_0^x u^{\alpha-1}(1-u)^{\beta-1}du, & for \quad 0 \le x \le 1 \\ 1, & for \quad x > 1; \end{cases} \tag{3}$$

which can be integrated directly. It also has the form of an incomplete beta function for which values for given values of α and β can be found from mathematical tables[18]. The incomplete beta function is usually denoted by $I_x(\alpha,\beta)$. If we write $F_x(x)$ with parameters α and β in

the form $F(x;\ \alpha,\beta)$, the correspondence between $I_x(\alpha,\beta)$ and $F(x;\ \alpha,\beta)$ is determined as follows, if $\alpha \geq \beta$, then
$F(x;\ \alpha,\beta) = I_x(\alpha,\beta)$ and if $\alpha < \beta$, then $F(x;\ \alpha,\beta)$ =1- $I_{(1-x)}(\beta,\alpha)$

2.8 Define Probability Limits on Activity durations

Suppose there are n (>2) arbitrary points in between the Optimistic and Pessimistic time.(these points can be found by Monte Carlo Simulation method that we present in 3.3) also , Let X be the proportion of times taking within the established limits[9]. Now pdf thus takes the form of Equation (1) and whose distribution will be beta distribution. After putting the values for n , r and s we able to find the positive value of pdf . Then by Probability for discrete data greater than the value of $(\alpha - cuts)$, we easily understand our desire interval[9] and this interval is 100% suitable for our operation times.

4. PROPOSED METHOD

The activity durations in project networks are usually difficult to estimate or determine exactly, it is reasonable to represent as fuzzy numbers. For this purposed our method follow the following stepwise sequence. Firstly, we take a prior estimate from an experienced Project Manager and adopt three operation times by using fuzzy Ranking methods. Secondly, identify the distribution for three times and applying the theory of probability to make three times appropriate for the distribution with the help of Monte Carlo simulation. Thirdly, Identification of Critical path and checking criticality of Activity and Path Network by Backward pass method by calculating FPERT .The basic idea is apply here a combination of concept of the α cuts, Zadeh's extension principle[7] and our proposed method. Finally, one numerical examples will be solved by the help of our proposed method, where some comparison will also be present for supporting our proposed method..The method proposed in this paper is described in the sequence presented in below. Our proposed method will follow two phase:
First Phase : Reliable Operation time Phase and
Second phase : Fuzzy PERT Calculation phase.

4.1 Reliable Operation time phase

Step 1:

In this step there needs a project completion table which is made by the prior knowledge or the previous experience of the project manager. From this table we collect the probable value of three operation times for further analysis.

Step 2:

Mean Time and Standard deviation of the activity time are calculated from previous data using fuzzy triangular distribution and Fuzzy Ranking method. Since fuzzy theory do not require any posterior frequency distribution. Here, for a triangular fuzzy numbers

$\tilde{T} = (O,\ M_0,\ P)$, the generalized mean value $G(\tilde{T})$ and Standard deviation $S(\tilde{T})$ are given by:

$G(\tilde{T}) = \dfrac{O + M_o + P}{3}$, and $S(\tilde{T}) = \dfrac{1}{18}[O^2 + M_o{}^2 + P^2 - O.M_o - O..P - M_o.P]$

where, $O =$ Optimistic time, $M_o =$ Most Optimistic time and $P =$ Pessimistic time

Step 3:

A pre-estimation table for FPERT analysis is followed containing optimistic, most optimistic and pessimistic time that would be modified for making the operation times more accurate and reliable ensuring a interval with establishing a confidence level.

Step 4:

To generate normally distributed realizations of activity durations, a two steps Monte Carlo simulation procedure[20] is used. Firstly, generate uniformly distributed random variables, u_i in $(0,1)$. General formula for random number generation can be of the form:

$u_i = \mathit{fractional\ \ part\ of}\ \left[(\pi + u_{i-1})^5\right]$

where, π and u_{i-1} was the previously generated seed number. This formula is a special case of the mixed congruential method of random number generation. Using the generated number normally distributed random numbers are generated applying the following equations[21].

$$x_k = \mu_x + s\sin t, \qquad \text{with}$$
$$s = \sigma_x\sqrt{-2\ln u_1}$$
$$t = 2\pi u_2$$

where, x_k is the normal realization, i.e. μ ; μ_x is the mean of x, σ_x is the standard deviation of x, and $u_1,\ u_2$ are the two uniformly distributed random variable realizations. Fuzzy triangular distribution is to use for calculating mean and variance.

Step 5:

Optimistic and pessimistic time are adjusted by applying fuzzy α cuts up to 0.5 (not more than) on optimistic and pessimistic time while most optimistic time remain same for fuzzy α cut properties. The wider the support of the membership function, the higher the uncertainty. The fuzzy set that contains all elements with a membership of $\alpha \in [0, 1]$. The higher the value of α, the higher the confidence in the parameter (Li & Vincent, 1995).

Step- 6:

In this section the three operation times will update and these times will be most accurate and reliable time for the project schedule. The table hold moderate value of O, M_O, P, μ and σ. Now will proceed Second phase.

4.2 Fuzzy PERT calculation phase

Step -7:

Earliest Start time, Earliest Finish time, Latest Starting time, Latest Finish time are calculated; and fuzzy backward pass method is to be use such that criticality of every activity in FPERT Network can measure.

Chen and Huang[16] used fuzzy numbers to express the operation times for all activities in a project network. They considered an activity i

Here we use some notation to identify each activity in the project network[7]. These are as,

$E\tilde{S}_i$ = earliest start time of the activity, $E\tilde{F}_i$ = earliest finish time of the activity

$L\tilde{S}_i$ = latest start time of the activity , $L\tilde{F}_i$ = latest finish time of the activity,

duration of each activity : d_j and Float time of the activity $F\tilde{T}_i$

The FPERT method using the update time calculation and moderate technique:

$E\tilde{S}_i = \max_{j \in P(i)} \{E\tilde{S}_j + d_j\}$ where $P(i)$ is the set of predecessors for activity i

and $E\tilde{F}_i = E\tilde{S}_i + d_j$

Again, $L\tilde{F}_i = \min_{j \in S(i)} \{L\tilde{F}_j - d_j\}$ also $L\tilde{S}_i = L\tilde{F}_J - d_i$

Step -8:

It is necessary to calculate Float time to identify the Critical Path of the project. Now by the proof of Chen-Tung Chen and Sue-Fen Huang[16] and applying following formula float time is calculated.

Float time, $F\tilde{T}_i = L\tilde{F}_i - E\tilde{F}_i$

Here we use the backward calculation which is more effective than the forward fuzzy method of calculation

Step -9:

In this case, calculate the criticality of each activity in the project network on the basis of Chen and Huang[16]. Let the fuzzy float time of activity i be $m_i = (O_i\ , M_{oi}, P_i)$, by making use of fuzzy estimators with compact support the critical activity is defined Chen and Huang. This is the criticality of each activity and it is a crisp number which result depends on the activity.

Therefore, we can write as:

$$C\tilde{T}_i = \begin{cases} 0 & if\ \ O_i \geq 0 \\ \dfrac{-O}{M_o - O} & if\ \ O_i < 0 < M_{oi} \\ 1 & if\ \ M_{oi} \leq 0 \end{cases}$$

Step -10:

Here we check the activity criticality on the FPERT network and find out the activity which criticality is 1 and which one is nearer to 1 or, takes other values. The formulation can be done by the following equation, Then the criticality of k^{th} path in the Network $\pi(P_K) = \min_{i \in p_k}\{C\tilde{T}_i\}$; where, P_K is the k^{th} path in the network. If Path P is critical then $\pi(P)$ must satisfy, $\pi(P) = \max_k \{\pi(P_k)\}$

This calculation will identify the activity which is on the critical path and which one is appropriate for activity shift when uncertainty arise in any project.

Step-11:

The activity which takes the value 1 will be the critical activity, and this activity will form the critical path for FPERT network. It will be most preferable route for the completion of any large scale project. Finally, we will present the critical Path and the probable critical Path for the large scale project. These series of Critical path ensure the percentage of completion of the project if uncertainty arises at the time of execution.

5. SIMULATION EXPERIMENTS AND PERFORMANCE INDICATORS

The purpose of this paper is to establish effective and reliable time estimation for a large scale project. In this context we basically take two steps- first of all, the purpose is to accurate the operation times of the project and secondly find out a critical path measuring the activity criticality on that path. The Numerical experiment calculated by our proposed method will show the effectiveness of the project.

5.1 Experimental design for Numerical Example

The network shown in Fig-1 was taken from the work of Konstantinos A. Chrysafis and Basil K.

Papadopoulos[7] with three operation time. The table for the operation times is as follows:

Ac	Activity time
1	(2.40, 4.54, 5.58)
2	(3.51, 5.74, 6.89)
3	(2.99, 4.54, 5.04)
4	(2.59, 3.68, 3.77)
5	(3.47, 4.43, 4.39)
6	(3.41, 4.86, 5.27)
7	(4.17, 5.22, 5.26)
8	(5.00, 6.27, 6.54)
9	(4.30, 5.67, 6.05)

Also, the network diagram for the project is as follows:

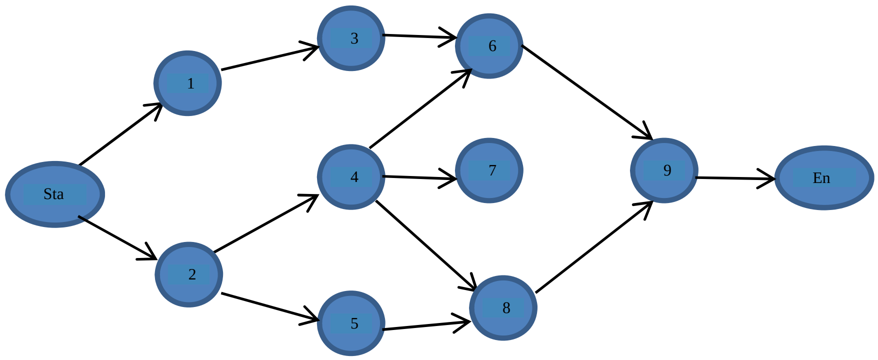

Figure 2 Network Diagram of Project

For meeting our purpose we now follow the sequential steps that explained in previous section of paper. The calculative results will be present here according to frame work of the total project.

5.2 Simulation results

The numerical result present here taking fractional value up-to two decimal places. These results are significant for measuring the criticality and Critical path of any large scale project.

Table 1: **Project Operation times and Calculation of Mean time and Standard deviation**

Activity	O	M_o	P	μ	σ
ST					
1	2.9	4.49	6.08	4.49	1.19
2	4.01	5.70	7.39	5.70	0.85
3	3.49	4.51	5.54	4.51	0.17
4	3.09	3.68	4.27	3.68	0.06
5	3.97	4.43	4.89	4.43	0.03
6	3.91	4.84	5.77	4.84	0.14
7	4.67	5.22	5.76	5.21	0.05
8	5.50	6.27	7.04	6.27	0.33
9	4.8	5.68	6.55	5.67	0.13
ED					

Table 2:Value of operation times after fuzzy α cuts

Activity	O	M_o	P	μ	σ
ST					
1	2.4	4.49	5.58	4.49	0.43
2	3.51	5.70	6.89	5.70	0.49
3	2.99	4.51	5.04	4.51	0.27
4	2.59	3.68	3.77	3.68	0.07
5	3.47	4.43	4.39	4.43	0.04
6	3.41	4.84	5.27	4.84	0.16
7	4.17	5.22	5.26	5.21	0.06
8	5.00	6.27	6.54	6.27	0.11
9	4.30	5.68	6.05	5.67	0.14
ED					

Table 3: Operation times adopted by Monte Carlo simulations ($u_0 = 0.5$ and $u_1 = 0.88$)

Activity	μ_x	σ	s	t	$s.\sin t$	μ
ST						
1	4.49	0.43	0.57	5.52	0.054	4.54
2	5.70	0.49	0.65	5.52	0.062	5.74
3	4.51	0.27	0.37	5.52	0.035	4.54
4	3.68	0.07	0.09	5.52	0.009	3.68
5	4.43	0.04	0.06	5.52	0.006	4.43
6	4.84	0.16	0.21	5.52	0.020	4.86
7	5.21	0.06	0.08	5.52	0.001	5.22
8	6.27	0.11	0.15	5.52	0.0002	6.27
9	5.67	0.14	0.19	5.52	0.0004	5.67
ED						

Table 4: Activity duration on Reliable Operation time phase

Activity	O	μ	P	σ
ST				
1	2.40	4.54	5.58	0.43
2	3.51	5.74	6.89	0.49
3	2.99	4.54	5.04	0.27
4	2.59	3.68	3.77	0.07
5	3.47	4.43	4.39	0.04
6	3.41	4.86	4.86	0.16
7	4.17	5.22	5.27	0.06
8	5.00	6.27	6.54	0.11
9	4.30	5.67	6.04	0.14
ED				

Table 5: Calculation of Earliest start time, Earliest finish time, Latest start time and Latest finish time

Ac	Activity time	$\widetilde{ES}_i$	$\widetilde{EF}_i$	$\widetilde{LS}_i$	$\widetilde{LF}_i$
1	(2.40, 4.54, 5.58)	(0, 0, 0)	(2.40, 4.54, 5.58)	(-5.66, 2.50, 10.77)	(-0.08, 7.04, 13.17)
2	(3.51, 5.74, 6.89)	(0, 0, 0)	(3.51, 5.74, 6.89)	(-7.59, -.0003, 7.59)	(-0.7, 5.74, 11.10)
3	(2.99, 4.54, 5.04)	(3.51, 5.74, 6.89)	(6.50, 10.29, 11.93)	(-0.08, 7.04, 13.17)	(4.96, 11.59, 16.16)
4	(2.59, 3.68, 3.77)	(3.51, 5.74, 6.89)	(6.10, 9.44, 10.66)	(-0.08, 6.49, 11.98)	(3.69, 10.18, 14.57)
5	(3.47, 4.43, 4.39)	(3.51, 5.74, 6.89)	(6.98, 10.18, 11.28)	(-0.7, 5.74, 11.10)	(3.69, 10.18, 14.57)
6	(3.41, 4.86, 5.27)	(6.50, 10.29, 11.93)	(9.91, 15.15, 17.20)	(4.96, 11.59, 16.16)	(10.23, 16.45, 19.57)
7	(4.17, 5.22, 5.26)	(6.10, 9.44, 10.66)	(10.27, 14.66, 15.92)	(4.97, 11.23, 15.40)	(10.23, 16.45, 19.57)
8	(5.00, 6.27, 6.54)	(6.98, 10.18, 11.28)	(11.98, 16.45, 17.82)	(3.69, 10.18, 14.57)	(10.23,16.45, 19.57)
9	(4.30, 5.67, 6.05)	(11.98,16.45, 17.82)	(16.28, 22.12, 23.87)	(10.23, 16.45, 19.57)	(16.28, 22.12, 23.87)

Table 6: Criticality and float time of each activity

Activity	Float time	Critical degree
1	(-5.66,2.50, 10.77)	0.69
2	(-7.59, 0, 7.59)	1
3	(-6.97, 1.30, 4.23)	0.84
4	(-6.97, 0.74, 3.91)	0.90
5	(-7.59, 0, 3.29)	1
6	(-6.97, 1.30, 2.37)	0.84
7	(-5.69, 1.79, 3.65)	0.76
8	(-7.59, 0, 1.75)	1
9	(-7.59, 0, 7.59)	1

Table 7: Now critical Path and Critical degree of each path:

Activity	Path	$\pi(P_k)$
1	$St \to 1 \to 3 \to 6 \to 9 \to ED$	0.69
2	$St \to 2 \to 4 \to 6 \to 9 \to ED$	0.84
3	$St \to 2 \to 4 \to 7 \to 9 \to ED$	0.90
4	$St \to 2 \to 4 \to 8 \to 9 \to ED$	0.84
5	$St \to 2 \to 5 \to 8 \to 9 \to ED$	1

Table 8: Observation and analysis

	Our proposed method		Method discuss in[17]		Percentage increases in proposed method
Path	Critical Path	$\pi(P_k)$	Critical Path	$\pi(P_k)$	
1	$St \to 1 \to 3 \to 6 \to 9 \to ED$	0.69	$St \to 1 \to 3 \to 6 \to 9 \to ED$	0.02	0.67
2	$St \to 2 \to 4 \to 6 \to 9 \to ED$	0.84	$St \to 4 \to 3 \to 6 \to 9 \to ED$	0.54	0.30
3	$St \to 2 \to 4 \to 7 \to 9 \to ED$	0.90	$St \to 2 \to 4 \to 7 \to 9 \to ED$	0.22	0.68
4	$St \to 2 \to 4 \to 8 \to 9 \to ED$	0.84	$St \to 2 \to 4 \to 8 \to 9 \to ED$	0.54	0.30
5	$St \to 2 \to 5 \to 8 \to 9 \to ED$	1	$St \to 2 \to 5 \to 8 \to 9 \to ED$	1	0.00

Possible Critical Path for Project Network

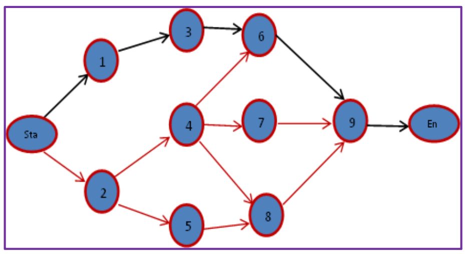

Fig 3: Critical Path (Path 1)

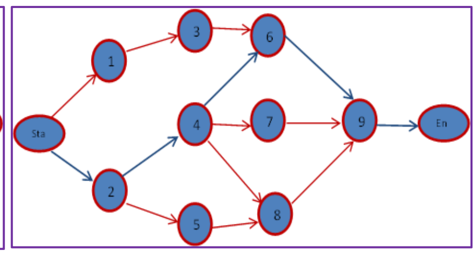

Fig 4: Critical Path (Path 2)

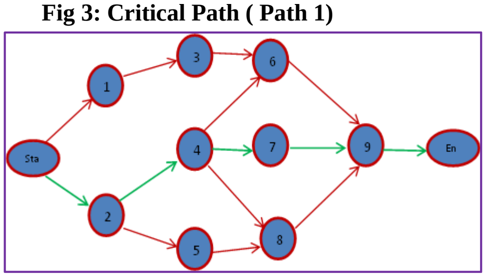

Fig 5: Critical Path (Path 3)

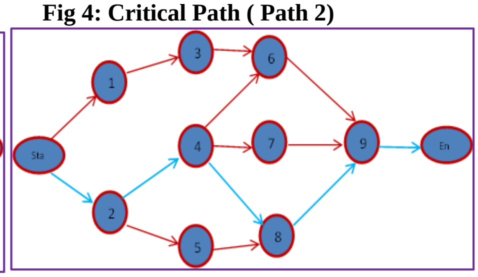

Fig 6: Critical Path (Path 4)

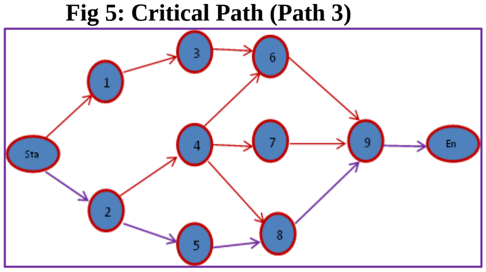

Fig 7: Critical Path (Path 5)

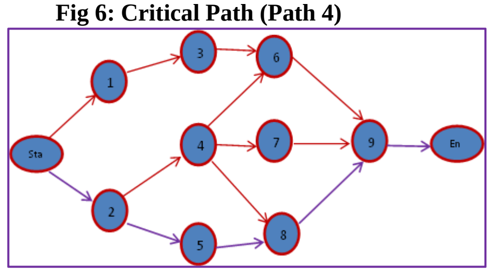

Fig 7: Critical Path of the FPERT

RESULTS ANALYSIS AND COMPARISON

The above table shows that, in both methods can clarify the critical path of the project and it is Path 5, but in the analysis of conventional FPERT method shows, the time required for completion of Project is 23.87 in our method, on the contrary method described in[7] takes 25.87 which is 8.34% more than our proposed method for Path 5. Planar will have the next option to take path 3 because of its greater path criticality. Moreover, Path 2 and Path 4 have the same degree of criticality whereas variance of Path 2 is greater than that of Path 4. So, in this case Project manager will take the decision to select Path 4 if any uncertainty arises to complete the project according to Path 3. The most optimal critical Path is Path 5 and Critical Path of the project is:

CONCLUSION

This paper shows an easy way to estimate more effective and accurate schedule for a large scale project by using Fuzzy activity analysis. It includes the process of choosing fuzzy field beta distribution and level of α cuts for any project with mean and variance. By increasing the accuracy of estimation, proposed method expressed the way of calculating three operation times more accurately. Moreover, the method discussed above shows an alternative technique of finding Critical Path by testing activity Criticality in the project network. It expresses the way to shift any Critical Path to another by shifting the critical path without crashing choosing. In addition to, our proposed method can reveals all possible Critical Path and their Criticality for the project completion. So it is easy to take any quick decision for project manager at any stage of project completion by measuring the criticality.

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